This exam covered some of the high school algebra that you must know how to do in order to be successful in precalculus. Students in the past who have scored less than 70% on this exam have succeeded in precalculus less than 5% of the time. So, if you had difficulty with this material you should talk to Professor Robertson about switching to PsTL 0722 section 3, which is a review of high school algebra.

1. [10] Find the area of this hockey rink. The width of the rink is 50 feet and the height is 20 feet. Use 3.14 for pi.

   \[ \text{ANS: Area} = \text{___________} \]

   \[
   \text{Total Area} = \text{Area}_{\text{circle}} + \text{Area}_{\text{rectangle}}
   \]
   \[= \pi r^2 + l \cdot w \]
   \[= \pi (10)^2 + 30 \cdot 20 \]
   \[= 3.14 \cdot 100 + 600 \]
   \[= 914 \]

2. [10] Simplify completely: \((x - 2)^2 - (x - 1)^2\)

   \[ \text{ANS: __________________________} \]

   \[
   (x - 2)^2 - (x - 1)^2 = x^2 - 4x + 4 - (x^2 - 2x + 1)
   \]
   \[= x^2 - 4x + 4 - x^2 + 2x - 1 \]
   \[= -2x + 3 \]

3. [10] Find the quotient and remainder of \(4x^4 - x + 1\) divided by \(x^2 + 1\).

   \[ \text{ANS: Quotient} = \text{___________} \]

   \[ \text{ANS: Remainder} = \text{___________} \]

   \[
   \begin{align*}
   \frac{4x^2 - 4}{x^2 + 1} \cdot & \quad 4x^4 + 0x^3 + 0x^2 - x + 1 \\
   - \left( \frac{4x^4 + 4x^2}{x^2 + 1} \right) \quad & \quad - 4x^2 - x + 1 \\
   & \quad - \left( \frac{4x^2 - 4}{x^2 + 1} \right) \quad - x + 5
   \end{align*}
   \]
4. [10] Factor completely: \( 12x^3 + 26x^2 - 10x \)  

ANS: ___________________________

First, factor out the GCF: \( 12x^3 + 26x^2 - 10x = 2x(6x^2 + 13x - 5) \)

Now, find two integers whose product is \( 6(-5) = -30 \) and whose sum is 13. They are 15 and -2. Replace the middle term with \( 15x - 2x \) and factor by grouping:

\[
12x^3 + 26x^2 - 10x = 2x(6x^2 + 15x - 2x - 5) = 2x(3x(2x + 5) - 1(2x + 5)) = 2x(2x + 5)(3x - 1)
\]

5. [10] Combine into a single fraction and simplify completely:

\[
\frac{2}{x^2 + 2x + 1} - \frac{1}{x^2 - 1}
\]

ANS: ___________________________

First, find the LCD.

\( x^2 + 2x + 1 = (x + 1)(x + 1) \)

\( x^2 - 1 = (x + 1)(x - 1) \)

The LCD has to contain each denominator as a factor. So, LCD = \( (x + 1)(x + 1)(x - 1) \).

Now, convert each fraction into an equivalent fraction but with the LCD as the denominator.

\[
\frac{2}{x^2 + 2x + 1} - \frac{1}{x^2 - 1} = \frac{2}{(x + 1)(x + 1)} \cdot \frac{x - 1}{x - 1} - \frac{1}{(x - 1)(x + 1)} \cdot \frac{(x + 1)}{(x + 1)} = \frac{2x - 2 - x + 1}{(x + 1)(x + 1)(x - 1)} = \frac{2x - 2 - x - 1}{(x + 1)(x + 1)(x - 1)} = \frac{x - 3}{(x + 1)(x + 1)(x - 1)}
\]
6. [10] Solve: \( \frac{3}{4}(x - 2) - 5 = \frac{x}{5} - 1 \)  
ANS: \( x = \) ________________________

Since this equation has fractions, we multiply each term by the LCD of the fractions (which is 20).

\[
\begin{align*}
\frac{3}{4}(x - 2) - 5 &= \frac{x}{5} - 1 \\
20 \left[ \frac{3}{4}(x - 2) \right] - 20[5] &= 20 \left[ \frac{x}{5} \right] - 20[1] \\
15(x - 2) - 100 &= 4x - 20 \\
15x - 30 - 100 &= 4x - 20 \\
15x - 130 &= 4x - 20 \\
11x - 130 &= -20 \\
11x &= 110 \\
x &= 10
\end{align*}
\]

7. [10] Simplify completely: \( \sqrt[3]{81x^8y^9} \)  
ANS: ______________________________

\[
\begin{align*}
\sqrt[3]{81x^8y^9} &= \sqrt[3]{3^4x^8y^9} \\
&= 3x^2y^3 \sqrt[3]{3x^2}
\end{align*}
\]

8. [10] Write in \( a + bi \) form: \( (2 - i) \div (3 - 2i) \)  
ANS: ______________________________

\[
\begin{align*}
(2 - i) \div (3 - 2i) &= \frac{2 - i}{3 - 2i} \\
&= \frac{2 - i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i} \\
&= \frac{6 + 4i - 3i - 2i^2}{9 + 6i - 6i - 4i^2} \\
&= \frac{6 + i - 2(-1)}{13} = \frac{8 + i}{13} = \frac{8}{13} + \frac{1}{13}i
\end{align*}
\]

9. [10] How much acid with a concentration of 25% must be added to acid with a 40% concentration to produce 50 gallons of a 30% solution?  
ANS: gal of 25% soln = _____________

Let \( x \) = gallons of 25% solution.  
Then, \( 50 - x \) = gallons of 40% solution

\[
\begin{align*}
\text{Acid in 25% solution} + \text{Acid in 40% solution} &= \text{Acid in 30% solution} \\
0.25x + 0.40(50 - x) &= 0.30(50) \\
0.25x + 20 - 0.40x &= 15 \\
-0.15x &= -5 \\
x &\approx 33.3
\end{align*}
\]

This rounds off to about 33 gallons.

10. [10] What number should be added to \( x^2 - 12x \) to complete the square?  
ANS: ______________________________

\[
\left[ \frac{-12}{2} \right]^2 = (-6)^2 = 36
\]